

# MULTIPLE COOPERATING MANIPULATORS: THE CASE OF KINEMATICALLY REDUNDANT ARMS

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## Abstract

This paper continues and extends existing work concerning two or more manipulators simultaneously grasping and transferring a common load. We specifically consider the case of one or more arms being kinematically redundant. Some existing results in the modeling and control of single redundant arms and multiple manipulators are reviewed. The cooperating situation is modeled in terms of a set of coordinates representing object motion and internal object squeezing. Nominal trajectories in these coordinates are produced via actuator load distribution algorithms introduced previously. A controller is developed to track these desired object trajectories while making use of the kinematic redundancy to additionally aid the cooperation and coordination of the system. It is shown how the existence of kinematic redundancy within the system may be used to enhance the degree of cooperation achievable.

## 1. INTRODUCTION

Recently, there has been increasing interest and research on the subject of multiple manipulators cooperating on a single task. This seems a natural extension of the notion of single manipulators performing tasks, and interest stems in no small part from NASA's declared interest in such systems for servicing the planned space station [12] and other extraterrestrial tasks.

Indeed, it may be in space that practical coordinated multi-arm systems find their first useful deployment. NASA has already identified a number of areas in which dual or multi-arm systems may be exploited in the space station, including servicing, assembly and construction, and launch, retrieval and handling of payloads [27, p.34]. In this paper, we consider the loading and control problems of such systems, to include both nonredundant and redundant arms.

Although the first space multi-arm systems will probably be nonredundant, it is felt that the current interest and research in single redundant arms (for some examples, see [4], [15], [23]) should, in a reasonably short time period, make the natural advantages of redundant arms (increased maneuverability, obstacle avoidance, subtask performance, etc.) available. It is natural to expect these systems to be exploited in the types of space-based applications mentioned above.

Some initial analysis of redundant arms in cooperating systems has already been made [8]-[10], [16], [21]. Much other work in various aspects of multi-arm systems has appeared recently, in modeling [5], [6], [15], [19], load distribution [20]-[22], [26], grasping [3], [13], [25], and control [1], [2], [7]-[9], [11], [14], [17], [18], [24]. In this paper we consider the case of  $L$  cooperating arms. In particular, we extend the work of Uchiyama and Dauchez [18], which considers two nonredundant arms, to  $L$  (possibly redundant) arms,

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employing coordinates specifying motion of the commonly held object and internal squeezing forces, which cause no motion but contribute to internal stresses in the system. The case of redundant arms is included by extending these coordinates.

A control structure is developed using the method of feedback linearization, to control the system in terms of the coordinates previously developed. This is similar in spirit to the work of Kreutz and Lokshin [9], Li, Hsu and Sastry [11], and Wen and Kreutz [24]. However, in contrast to [9] and [24], internal forces may be controlled and are represented directly at the common object coordinate level, as opposed to [11], where they are represented at the end effector level.

The paper is organized as follows. Preliminaries and some results on load distribution are presented in Section 2. Section 3 contains the necessary coordinate transformations and control structure in the nonredundant case, and this is extended to include redundant arms in Section 4. Conclusions are presented in Section 5.

## 2. LOAD DISTRIBUTION

When two (or more) manipulators form a closed chain (or chains) in cooperatively grasping a common object, the mobility of the resulting system will usually be lower than the total number of actuators available in the arms. This overabundance of kinematically dependent inputs arises from the kinematic constraints necessary to maintain the closed chain(s). If it is desired to utilize  $R$  actuators, where  $R$  is greater than the system mobility, then the loading in the system is not uniquely specified by a trajectory of the commonly grasped object. This means that in controlling such systems, a choice must be made, either implicitly or explicitly, to select one from a number of possible loading states at each point along the trajectory.

Clearly, since different selections will give rise to different internal loadings and actuator demands, some loading states will be more desirable than others — indeed, some may be unacceptable in practice, requiring unachievable actuator loads or imposing unsustainable internal forces on the manipulators and/or commonly held object. Investigation of these issues has resulted in a number of suggested solutions of the load distribution problem ([1], [7]-[9], [13], [14], [16], [18], [20], [21], [24]-[26]). In these, the common emphasis is on specifying end-effector forces/moments throughout the trajectory, subject to criteria involving internal object forces or joint torque requirements.

For this, the motion equations for the commonly held object are developed using the Newton and Euler equations

$$\underline{f} = m\ddot{\underline{r}} \quad (2.1)$$

$$\underline{n} = I\dot{\underline{\omega}} + \underline{\omega} \times (I\underline{\omega}) \quad (2.2)$$

where  $m$  and  $I$  are the mass and inertia matrix of the object, and  $\underline{r}$  and  $\underline{\omega}$  are the position of the center of mass of the object (expressed in a global reference frame) and the angular velocity of the object, respectively.

Assuming the rigid object is rigidly grasped by  $L$  robotic arms (or fingers), and that the arms may impart forces and moments to the object,  $\underline{f}$  and  $\underline{n}$  are given by

$$\underline{f} = \sum_{i=1}^L \underline{f}_i + m\underline{g} \quad (2.3)$$

$$\underline{n} = \sum_{i=1}^L \underline{n}_i + \sum_{i=1}^L \underline{s}_i \times \underline{f}_i \quad (2.4)$$

where  $\underline{f}_i$ ,  $\underline{n}_i$  are the forces and moments applied to the object by the  $i^{th}$  end effector, respectively,  $\underline{g}$  is the gravity vector, and  $\underline{s}_i$  is the position of the  $i^{th}$  end effector with respect to the object coordinates associated with  $\underline{f}$  and  $\underline{n}$  (see Fig. 1).

Equations (2.1)-(2.4) may be combined to yield

$$\underline{P} = W \underline{F} \quad (2.5)$$

where

$$\underline{P} = R \ddot{\underline{\phi}} + \underline{Q}$$

is the generalized load required to move the object, with

$$R = \begin{bmatrix} mI_u & 0 \\ 0 & I \end{bmatrix}$$

$$\ddot{\underline{\phi}} = [\ddot{\underline{r}}^T \quad \ddot{\underline{\omega}}^T]^T$$

$$\underline{Q} = \begin{bmatrix} -m\underline{g} \\ \underline{\omega} \times (I\underline{\omega}) \end{bmatrix}$$

and the loads being imparted by the  $L$  arms are

$$\underline{F} = [\underline{f}_1^T, \underline{n}_1^T, \dots, \underline{f}_L^T, \underline{n}_L^T]^T$$

with

$$W = \begin{bmatrix} I_v & 0 & \dots & I_v & 0 \\ S_1 & I_v & \dots & S_L & I_v \end{bmatrix}$$

$$S_i = \begin{bmatrix} 0 & -S_{i3} & S_{i2} \\ S_{i3} & 0 & -S_{i1} \\ -S_{i2} & S_{i1} & 0 \end{bmatrix}$$

$$\underline{s}_i = [S_{i1} \quad S_{i2} \quad S_{i3}]^T$$

$I_v$  is the  $v \times v$  identity matrix (the dimension of  $v$  is determined by the particular problem being solved, e.g.  $u = v = 3$  in the spatial case).

We have multiple solutions in general for the end effector forces  $\underline{F}$ , given the desired object motion represented by  $\underline{P}$ . The general solution to (2.5) is given by

$$\underline{F} = W^+ \underline{P} + (I_l - W^+ W) \underline{\epsilon} \quad (2.6)$$

where  $W^+ \in \mathbb{R}^{l \times k}$  (where  $k$  and  $l$  are the dimensions of  $\underline{P}$  and  $\underline{F}$  respectively) is a generalized inverse, or pseudoinverse of  $W$  and may be defined in general by  $W^+ = AW^T(WAW^T)^{-1}$  for some positive definite  $A$ . In (2.6),  $\underline{\epsilon} \in \mathbb{R}^l$  is an arbitrary vector whose choice determines which of the possible solutions of (2.5) is chosen.

Notice that choosing an  $\underline{\epsilon}$  in (2.6) represents one method of load distribution, from the object load  $\underline{P}$  to end effector loads  $\underline{F}$ , which for nonredundant arms uniquely specifies the joint torques (the redundant case is discussed in Section 4). Various authors, e.g. [13], [25], have suggested schemes to choose  $\underline{\epsilon}$  in (2.6) subject to object-related criteria, such as 'squeezing' effects on the object. However, care must be taken in such approaches to allow for the inherent squeezing effects in (2.6) due to the first term  $W^+ \underline{P}$ . The contributions to internal forces by this term (which include unavoidable inertial loadings) are investigated and quantified in [22].

Alternatively, by looking at the effects  $\underline{F}_i$  of end effector forces  $\underline{F}_i$  at the object coordinates (where  $\underline{F} = [\underline{F}_1^T, \dots, \underline{F}_L^T]^T$  and  $\underline{F}_i = [f_i^T, \underline{n}_i^T]^T$ ), we have

$$\underline{P} = [I_1 \quad \dots \quad I_L] \underline{F} := \overline{W} \underline{F} \quad (2.7)$$

which is clearly similar in form to (2.5). At these coordinates, an equation similar to (2.6) may be formed, and load distribution made at this level. This is the impetus of several approaches ([1], [7], [8], [14], [18], [26]).

The appropriate transformation in obtaining (2.7) is

$$\bar{F}_i = \begin{bmatrix} I & 0 \\ S_i & I \end{bmatrix} F_i = W_i F_i, \quad (2.8)$$

the use of which will be exploited further in Section 3.

A different strategy for load distribution is to consider the manipulator dynamics of the cooperating system, which may be expressed as (see [5], [6], [21], for full details of the notation used here)

$$\tau_\beta = [M(\beta)]\ddot{\beta} + \dot{\beta}^T [N(\beta)]\dot{\beta} - J_\beta^T(\beta)F \quad (2.9)$$

(where  $\beta$  represents the joint variables of the  $L$  manipulators,  $M$  is the effective inertia matrix,  $N$  involves the centripetal and coriolis effects,  $J_\beta$  is the Jacobian relating end effector variables to joint variables  $\beta$  and  $\tau_\beta$  is the force/torque control vector corresponding to  $\beta$ ).

Consideration of load distribution effects at this (joint) level allows actuator loading effects to be taken into account, in contrast to the purely object-based criteria mentioned above. Work in this area has been performed by the authors [20], [21] using (2.6) combined with (2.9) to use joint torque based criteria to select  $\underline{\epsilon}$ , hence loading throughout the system. Joint space criteria are also used in [8], [14], [26], in which (2.7) is utilized in placed of (2.6).

However, for such joint-based methods, it is necessary to monitor internal forces built up in the object as a result of the distribution strategy applied. Motion which alleviates actuator demands will not be successful if internal forces built up destroy the object transported (of course the opposite logic applies to object based methods, where end effector forces calculated must be attainable by realistic actuator torque levels). The work in [8], [26] utilizes joint space criteria subject to

$$\bar{F}_i = \alpha_i \underline{P} \quad (2.10)$$

where  $\sum_{i=1}^L \alpha_i = 1$ , and each  $\alpha_i$  is a non-negative scalar. This approach apportions only motion-causing components to  $\bar{F}_i$ ; thus, only the unavoidable inertial forces referred to earlier will build up in the object. However, this excludes examples such as in the two-arm case,  $\alpha_1 = -1$  and  $\alpha_2 = 2$ , which will also produce the desired object motion  $\underline{P}$ . In other words, (2.10) requires all arms to be ‘pulling in the same direction.’ This may not always be desirable for the individual arms, and indeed, in some cases it may be desirable to have forces propagated through the object (for example, to allow one arm to assist in the motion of another in certain configurations where motion in a given direction is more readily obtained by the actuator load state of that arm — notice again that modeling this situation is inherently joint-based).

Notice also that, again for the two-arm case, we have two distinct situations: (a)  $\{0 \leq \alpha_1 \leq 1 \text{ and } \alpha_2 = 1 - \alpha_1\}$ ; or (b)  $\{(\alpha_1 < 0, \alpha_2 = 1 - \alpha_1) \text{ or } (\alpha_2 < 0, \alpha_1 = 1 - \alpha_2)\}$ , which correspond to ‘pulling together’ or ‘squeezing,’ as discussed above. In both cases the arms ‘cooperate’ in the sense that the desired object motion is obtained, but only the first case represents load sharing in the sense of (2.10). Such issues should be considered in those cases, such as [14], when bias squeezing effects are added to solutions developed on the basis of (2.10).

In the case of the methods of [20], [21], object loading again needs to be regulated. This issue is addressed in [22], and the case of combined joint/object space distribution methods is the subject of current research. In the following section we consider the problem of control, and propose a methodology for controlling object motion and internal object squeezing applicable to the types of load distribution discussed above. This is extended to the case of redundant arms in Section 4.

### 3. OBJECT COORDINATE CONTROL

For the case of multiple manipulators, various control schemes have already been proposed of the feedback linearization, or 'computed torque,' type. Various philosophies of control synthesis have been proposed, including the use of LQ theory [17] and various PD or PID strategies. Of the second type, methods differ mainly in the types of coordinates to be controlled, from joint coordinates [14], [24] to end effector or operational space coordinates [8], to generalized object coordinates [3], [7], [9], [11]. In this section, we propose a controller of the computed torque type based on coordinates related directly to object motion and squeezing.

The goal is to obtain a control structure at the object level in which various strategies may be implemented. The coordinate system established here allows direct access to the force and motion coordinates involved in internal object squeezing. It is felt that nominal trajectories in these coordinates will, in many cases, provide the most clear description of the cooperating arm situation. For example, depending on the held object and task, a particular internal force strategy (perhaps no internal squeezing in given directions) may be mandatory, and this type of situation is clearly described in terms of the coordinate model introduced below.

Alternatively, nominal trajectories may be established by various other means. For example, load distribution algorithms of the type described in the previous section result in desired loadings for  $\underline{\tau}$ ,  $\underline{E}$ , or  $\underline{\bar{E}}$ , and this is easily converted to desired loadings in our object coordinates (for a discussion of this, and examples of its importance in load distribution methods, see [22]). The control structure developed in this section allows the adoption of the above, and other types of methodologies, in a straightforward approach to object space control.

First, we must establish the relevant coordinate transformations necessary. The coordinates to be controlled are the object motion coordinates  $\underline{\phi}$  in (2.5), together with coordinates  $\Delta \underline{z}$  to be defined, where  $\Delta \underline{z} \in \mathbb{R}^{6(L-1)}$  in general, and represent relative small displacements between end effector motions referenced at the generalized object coordinates. For the case of two nonredundant arms, the transformations have been developed by Uchiyama and Dauchez [18] who applied a hybrid position/force scheme to the result. We extend the result here to cover  $L$  arms.

We have seen in (2.8) how end effector forces/moments  $\underline{F}_i$  may be related to their equivalent effects  $\underline{\bar{F}}_i$  at the object coordinates. Define by  $\underline{z}_i$  the coordinates of the frame obtained by translating the end effector frame of arm  $i$  by  $\underline{g}_i$ , i.e. the frame associated with  $\underline{\bar{F}}_i$ . Then by the duality of force/velocity relationships we have

$$\dot{\underline{z}}_i = W_i^T \dot{\underline{z}}_i \quad (3.1)$$

with  $\underline{z}_i$  the end effector coordinates of arm  $i$ , and  $W_i$  is constant, nonsingular and given by (2.8).

Next we solve (2.7) for  $\underline{\bar{F}}$  in terms of  $\underline{P}$  and coordinates  $\underline{P}_r$ , where  $\underline{P}_r$  parameterize the internal object forces, by

$$\underline{\bar{F}} = U \begin{bmatrix} \underline{P} \\ \underline{P}_r \end{bmatrix} = \begin{bmatrix} \bar{W}^+ & V \end{bmatrix} \begin{bmatrix} \underline{P} \\ \underline{P}_r \end{bmatrix} \quad (3.2)$$

with  $U \in \mathbb{R}^{6L \times 6L}$ ,  $\underline{P}_r \in \mathbb{R}^{6(L-1)}$ ,  $\bar{W}^+ \in \mathbb{R}^{6L \times 6}$ ,  $V \in \mathbb{R}^{6L \times 6(L-1)}$ , and  $\bar{W}^+$  is given by  $A\bar{W}^T(\bar{W}A\bar{W}^T)^{-1}$ , with  $A \in \mathbb{R}^{6L \times 6L}$  invertible, and

$$V = \begin{bmatrix} I & 0 & 0 & \dots & 0 \\ -I & I & 0 & & \vdots \\ 0 & -I & I & & \vdots \\ 0 & 0 & \ddots & \ddots & 0 \\ \vdots & & 0 & -I & I \\ 0 & & & 0 & -I \end{bmatrix} \quad (3.3)$$

In (3.3),  $I$  represents the  $6 \times 6$  identity matrix. From (3.2) and (3.3), it may be readily seen that  $V$  is a parametrization of the nullspace of  $\bar{W}$  (note  $\bar{W}V = 0$ ).  $\underline{P}_r$  represents the  $(L - 1)$  independent squeezing forces between  $(L - 1)$  distinct end effector pairs due to the construction of  $V$ , i.e. components of  $\underline{F}$  which do not contribute to motion but squeeze the object. Furthermore,  $U$  in (3.2) is nonsingular and constant if  $A$  is constant. It is a simple exercise to check that, for

$$A = \begin{bmatrix} A_1 & & 0 \\ & \ddots & \\ 0 & & A_L \end{bmatrix} \quad \text{with} \quad A_i = \begin{bmatrix} \alpha_1^i & & 0 \\ & \ddots & \\ 0 & & \alpha_6^i \end{bmatrix} \quad (3.4)$$

with  $\alpha_j^i$  positive and  $\sum_{j=1}^6 \alpha_j^i = 1$ , then in (3.2) we obtain, with  $\underline{P}_r = 0$ ,  $(\underline{F}_i)_j = \alpha_j^i(\underline{P})_j$ , i.e. we may do load sharing in the ‘pulling together’ sense of (2.10) as before when  $\underline{P}_r = 0$ . This extends the work in [18], where  $A = I$  and  $V$  was simpler for the two arm case.

Again using the force/velocity duality and defining  $\underline{z} = [\underline{z}_1^T, \dots, \underline{z}_L^T]^T$ , we have from (3.2)

$$\begin{bmatrix} \dot{\underline{\phi}} \\ \Delta \dot{\underline{z}} \end{bmatrix} = U^T \dot{\underline{z}} \quad (3.5)$$

where  $\Delta \dot{\underline{z}}$  is the difference in velocities between the frames represented by the  $\underline{z}_i$ ’s, i.e. the velocities associated with  $\underline{P}_r$ .

We may apply the ‘computed torque’ methodology to the system in coordinates  $[\dot{\underline{\phi}}^T \Delta \dot{\underline{z}}^T]^T$  in (3.5). Specifically, referring to (2.9), (3.1) and (3.5), if the second order kinematics of the arms are given by

$$\ddot{\underline{x}} = J_\beta(\underline{\beta})\ddot{\underline{\beta}} + \dot{\underline{\beta}}^T [H_\beta(\underline{\beta})] \dot{\underline{\beta}} \quad (3.6)$$

then the input of

$$\underline{\tau}_\beta = [M][J]^{-1}[\tilde{W}]^T[U^T]^{-1}\underline{w} + \dot{\underline{\beta}}^T[N]\dot{\underline{\beta}} - J^T \underline{F} - [M][J]^{-1}\dot{\underline{\beta}}^T[H]\dot{\underline{\beta}} \quad (3.7)$$

sets the dynamics (assuming perfect modeling, measurements, etc.) to

$$\begin{bmatrix} \ddot{\underline{\phi}} \\ \Delta \ddot{\underline{z}} \end{bmatrix} = \underline{w} \quad (3.8)$$

(in (3.7),  $\tilde{W} = \begin{bmatrix} W_1 & & 0 \\ & \ddots & \\ 0 & & W_L \end{bmatrix}$ , and it is assumed that  $U$  is constant, although this is not necessary).

Notice that in (3.7) the dependence of  $M$ ,  $J$ ,  $N$  and  $H$  on  $\underline{\beta}$  has been omitted.

Any one of a number of possible strategies may be used to set  $\underline{w}$  (i.e. PD, PID, force feedback). Issues associated with this type of control are well documented ([24] contains a discussion of such issues for joint space and end effector PD control of cooperating arms). It is believed by the authors that the control structure represented by (3.7) is particularly appealing since the coordinates involved go directly to the ‘heart’ of the cooperating arm problem, namely those associated with the object, its motion and its loading which, as we have seen, may be related directly to the overall system loading. The extension at the object coordinates to include  $\underline{P}_r$  allows this to be introduced. This differs from [9] where a feedforward approach is used to control internal forces after system linearization. It also represents a generalization of the framework of [11], in which object motion is controlled together with non-motion causing end effector force components. In (3.8), the extension of coordinates allows the consideration of a wide class of strategies at the object level, due to the exposure of the object motion/squeezing structure represented by (3.8) (and

in fact  $\underline{w}$  may be chosen in (3.8) to implement a strategy similar to that in [11], but with the internal forces controlled being  $\underline{P}_r$  instead of end effector force components). Of course, as in all such approaches, effectiveness will be limited by sensory data and computational power available, and for some cases (e.g. where some parameters are not well known) other methods, such as adaptive control methods, will prove useful. However, it is believed irrespectively that further investigation of the control scheme introduced here ((3.7)) will prove both informative and useful. Results will be presented in a future paper.

#### 4. REDUNDANT ARMS IN COOPERATION

If one or more of the arms in the system is kinematically redundant, we are presented with additional problems due to the increased complexity — for the redundant arm(s), we have an overabundance of kinematically independent inputs relative to the end effector(s) of the arm(s). This means that we have multiple solutions for the inverse kinematics, the investigation of which is a large research area of its own; see, for example, [4], [15], [23]. Additionally, the object/end effector spaces do not specify the system totally. Recall in Section 3 we had  $6L$  convenient variables for the  $6L$  actuators — 6 coordinates of object motion, and  $(6 - 1)L$  independent velocity differences between end effector related frames. In the redundant situation, these coordinates remain but now there are  $6L + R$  actuators where  $R$  is the total degree of redundancy in the arms. To control the system we need to specify  $R$  more (independent) variables, representing the choice of arm configurations to be made.

However, the potential benefits of using redundant arms are huge — the self-motion of the arms may be used to avoid collisions with obstacles (and each other), and also the system as a whole gains more mobility, which may be used to reduce actuator demands and aid in cooperative movement. First results in trajectory planning for multiple cooperating arms are presented in [10] and [16], and a generalized inverse of the Jacobian is used in [8] to reduce torque demands while controlling object motion forces.

Here we develop a controller by extending the coordinate space to specify  $R$  extra variables to be controlled independently using the redundant actuator(s). The idea is to make these introduced variables significant to the multiple manipulator problem. Notice that one advantage is that all results related to end effector, or object spaces, still remain pertinent — the kinematic redundancy is ‘decoupled’ from the object. Therefore the  $R$  extra coordinates should be directly associated with the redundant arm(s) (this is clear when it is remembered that their purpose is to specify the configuration(s) of the arm(s)).

This means that we must specify  $R$  coordinates  $\underline{r}$  such that  $\underline{r}$  is independent of  $\underline{\phi}$  and  $\Delta \underline{z}$  (equivalently independent of the end effector coordinates) and that  $\underline{r}$  and  $\underline{\phi}$  (equivalently  $\underline{r}$  and  $\underline{x}$ ) uniquely define the configurations of all arms. Mathematically, we must have  $R$  constraints such that

$$\dot{\underline{r}} = C \dot{\underline{\beta}} \quad (4.1)$$

where  $C \in \mathbb{R}^{R \times (6L+R)}$ , and  $C$  is such that  $B \in \mathbb{R}^{(6L+R) \times (6L+R)}$ , defined by

$$B = \begin{bmatrix} [J] \\ [C] \end{bmatrix} \quad (4.2)$$

is invertible. This will be true provided  $\underline{r}$  is chosen correctly and  $J$  is of full rank (i.e. no arm is in a singular configuration).

Given a choice of  $\underline{r}$ , and noting (4.2), an extension of the control in Section 3 leads to

$$\underline{\tau}_\beta = [M] B^{-1} W^* [\overline{U}^T]^{-1} \underline{w} + \dot{\underline{\beta}}^T [N] \dot{\underline{\beta}} - J^T \underline{F} - [M] B^{-1} \dot{\underline{\beta}}^T [\overline{H}] \dot{\underline{\beta}} \quad (4.3)$$

where  $W^* = \begin{bmatrix} \dot{W} & 0 \\ 0 & I_R \end{bmatrix}$ ,  $\overline{U}^T = \begin{bmatrix} U^T & 0 \\ 0 & I_R \end{bmatrix}$ ,  $\dot{\underline{\beta}}^T [\overline{H}] \dot{\underline{\beta}} = \begin{bmatrix} J \\ C \end{bmatrix} \dot{\underline{\beta}}$ , and  $I_R$  is the  $R \times R$  identity matrix) which sets the dynamics to

$$\begin{bmatrix} \ddot{\underline{\phi}} \\ \ddot{\Delta \underline{z}} \\ \ddot{\underline{r}} \end{bmatrix} = \underline{w} \quad (4.4)$$

The comments made in Section 3 apply again to this situation, and if the kinematics have been planned, giving the  $\underline{r}$  trajectories, various strategies may be used to select  $\underline{u}$  to track the trajectory as before. This strategy of extending the workspace coordinates to control redundant manipulators was used by Egeland [4] for single redundant manipulators, and (4.3)-(4.4) represents the extension of this to the multiple manipulator situation.

One further intriguing possibility is that of selecting  $\underline{r}$  trajectories 'on-line' to allow the redundant arms to configure themselves along the trajectory to react to problems unforeseen in the planning, while continuing to track the planned desired object motion and squeezing trajectories. Research in this direction is underway currently.

## 5. CONCLUSIONS

We have presented a framework for the control of multi-arm robot systems. Cooperation is modeled in terms of coordinates representing object motion and internal object squeezing. A control structure has been developed to track trajectories in these coordinates. This was extended to allow the use of redundant arms by making an appropriate extension of coordinates.

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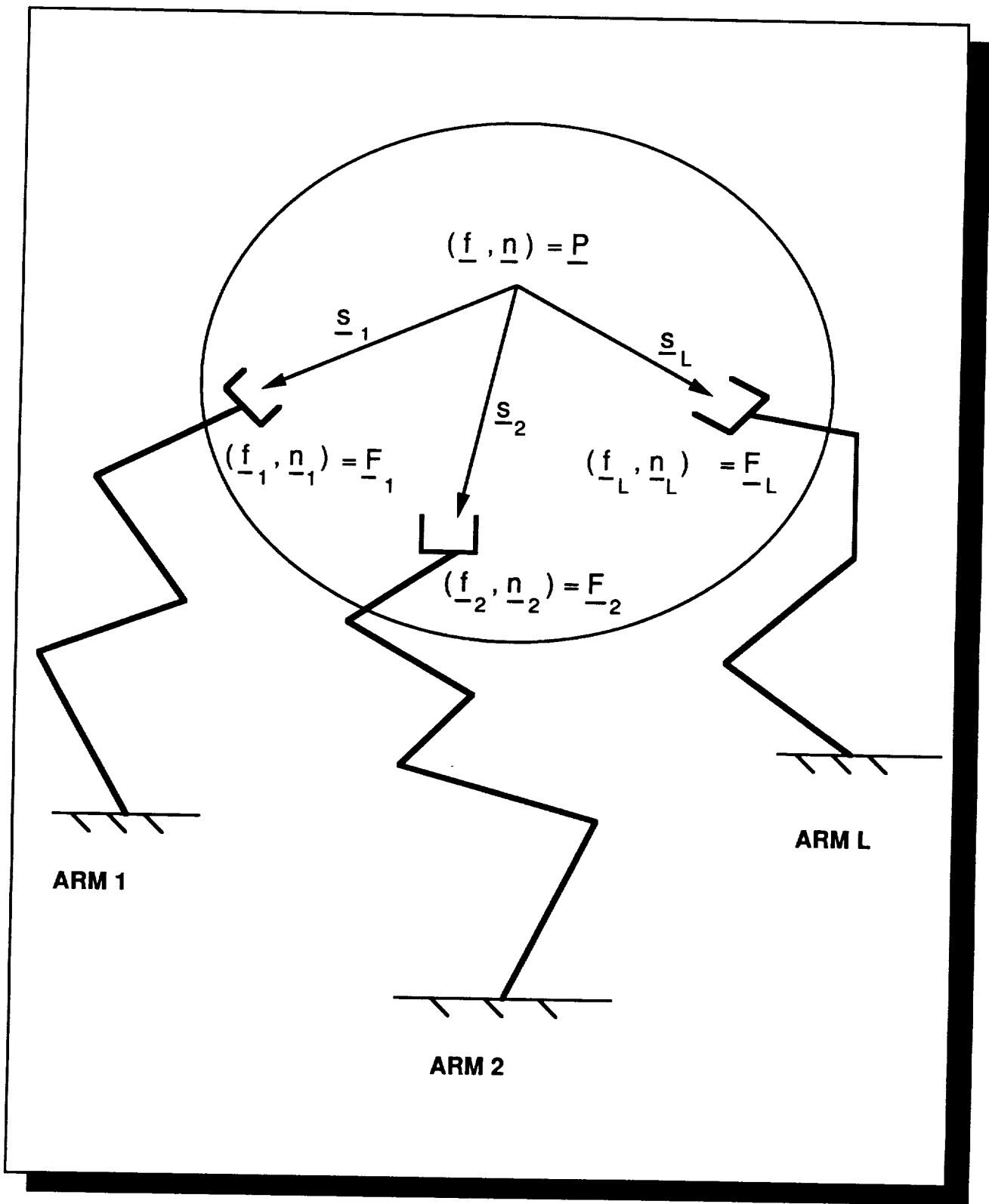


Figure 1. Multiple arm cooperating system.